



L4 – Quantum Mechanics 4

From Quantum to Classical

**What does happen to a
quantum state when it is
measured?**

A quantum state may follow two types of time evolution:

1. On its own, it evolves with the **Schrödinger equation** (unitary and deterministic)
2. During a measurement, it evolves with the **Born rule** (non-unitary and stochastic)

Brian Greene



Elise Crull

- Decoherence

Sean Carroll

- Many worlds
- Objective collapse

Carlo Rovelli

- Relational interpretation

Decoherence

Many worlds

Objective collapse

Relational interpretation

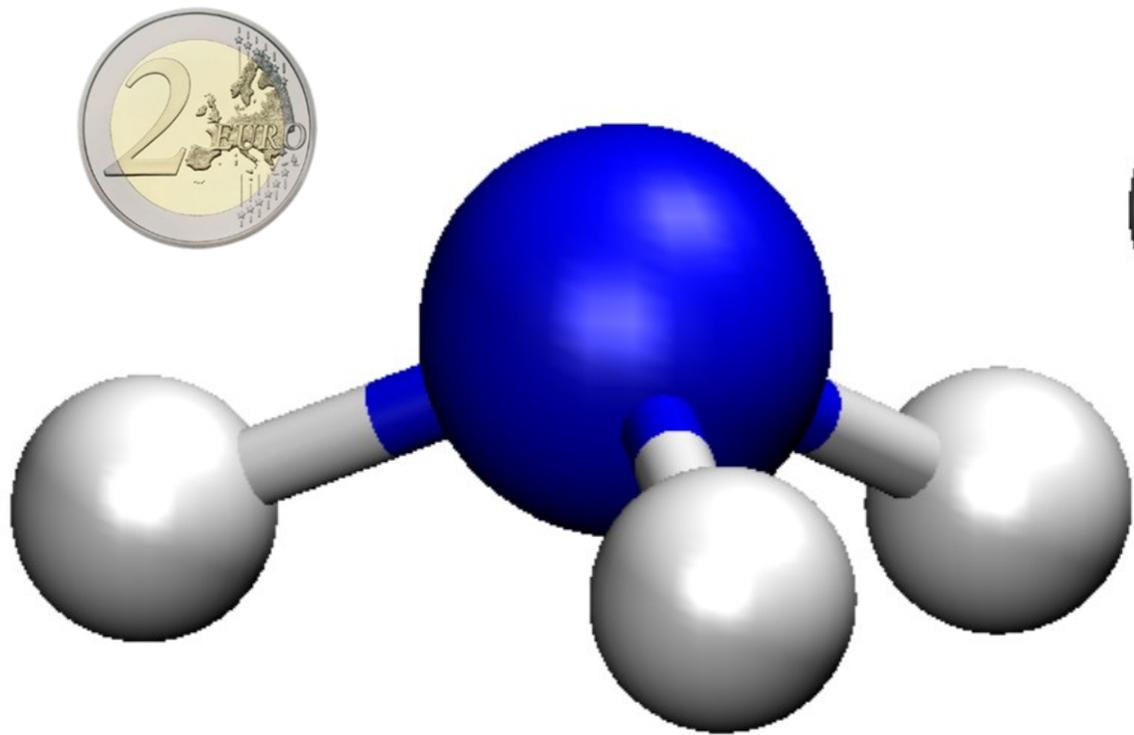
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Proposed Solutions to the Measurement Problem

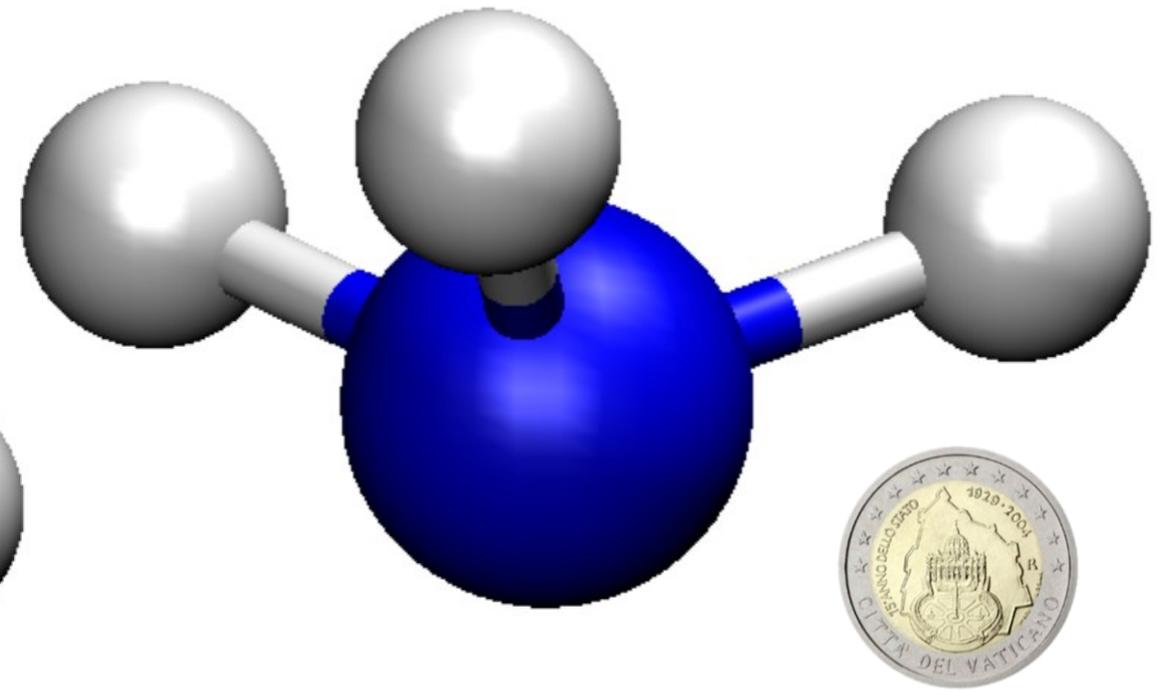
Interpretation	Year published	Author(s)	<u>Deterministic?</u>	<u>Ontic wave-function?</u>	Unique history?	<u>Hidden variables?</u>	<u>Collapsing wave-functions?</u>	Observer role?	<u>Local dynamics?</u>	<u>Counter-factually definite?</u>	<u>Extant universal wave-function?</u>
<u>Consciousness causes collapse</u>	1961–1993	<u>John von Neumann</u> , <u>Eugene Wigner</u> , <u>Henry Stapp</u>	No	Yes	Yes	No	Yes	Causal	No	No	Yes
<u>Consistent histories</u>	1984	<u>Robert B. Griffiths</u>	No	No	No	No	No	No	Yes	No	Yes
<u>Copenhagen interpretation</u>	1927–	<u>Niels Bohr</u> , <u>Werner Heisenberg</u>	No	Some	Yes	No	Some	No	Yes	No	No
<u>de Broglie–Bohm theory</u>	1927–1952	<u>Louis de Broglie</u> , <u>David Bohm</u>	Yes	Yes	Yes	Yes	Phenomenological	No	No	Yes	Yes
<u>Ensemble interpretation</u>	1926	<u>Max Born</u>	Agnostic	No	Yes	Agnostic	No	No	No	No	No
<u>Many-minds interpretation</u>	1970	<u>H. Dieter Zeh</u>	Yes	Yes	No	No	No	Interpretational	Yes	Ill-posed	Yes
<u>Many-worlds interpretation</u>	1957	<u>Hugh Everett</u>	Yes	Yes	No	No	No	No	Yes	Ill-posed	Yes
<u>Objective-collapse theories</u>	1986–1989	<u>Ghirardi–Rimini–Weber</u> , <u>Penrose interpretation</u>	No	Yes	Yes	No	Yes	No	No	No	No
<u>QBism</u>	2010	Christopher Fuchs, Rüdiger Schack	No	No	Agnostic	No	Yes	Intrinsic	Yes	No	No
<u>Quantum logic</u>	1936	<u>Garrett Birkhoff</u>	Agnostic	Agnostic	Yes	No	No	Interpretational	Agnostic	No	No
<u>Relational interpretation</u>	1994	<u>Carlo Rovelli</u>	No	No	Agnostic	No	Yes	Intrinsic	Possibly	No	No
Time-symmetric theories	1955	<u>Satosi Watanabe</u>	Yes	No	Yes	Yes	No	No	No	No	Yes
<u>Transactional interpretation</u>	1986	<u>John G. Cramer</u>	No	Yes	Yes	No	Yes	No	No	Yes	No

en.wikipedia.org/wiki/Interpretations_of_quantum_mechanics

Stating the Measurement Problem



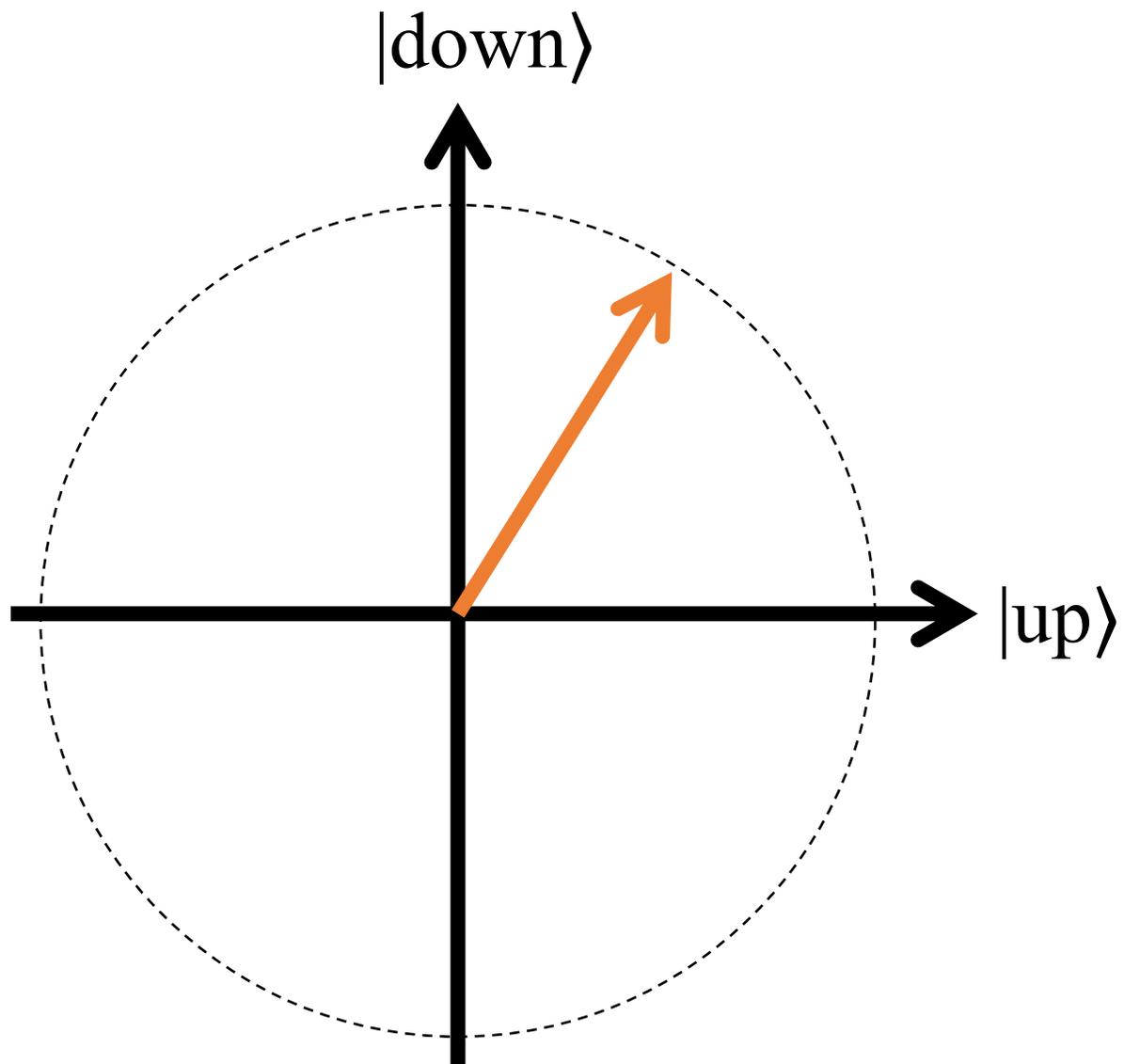
$|\text{up}\rangle$

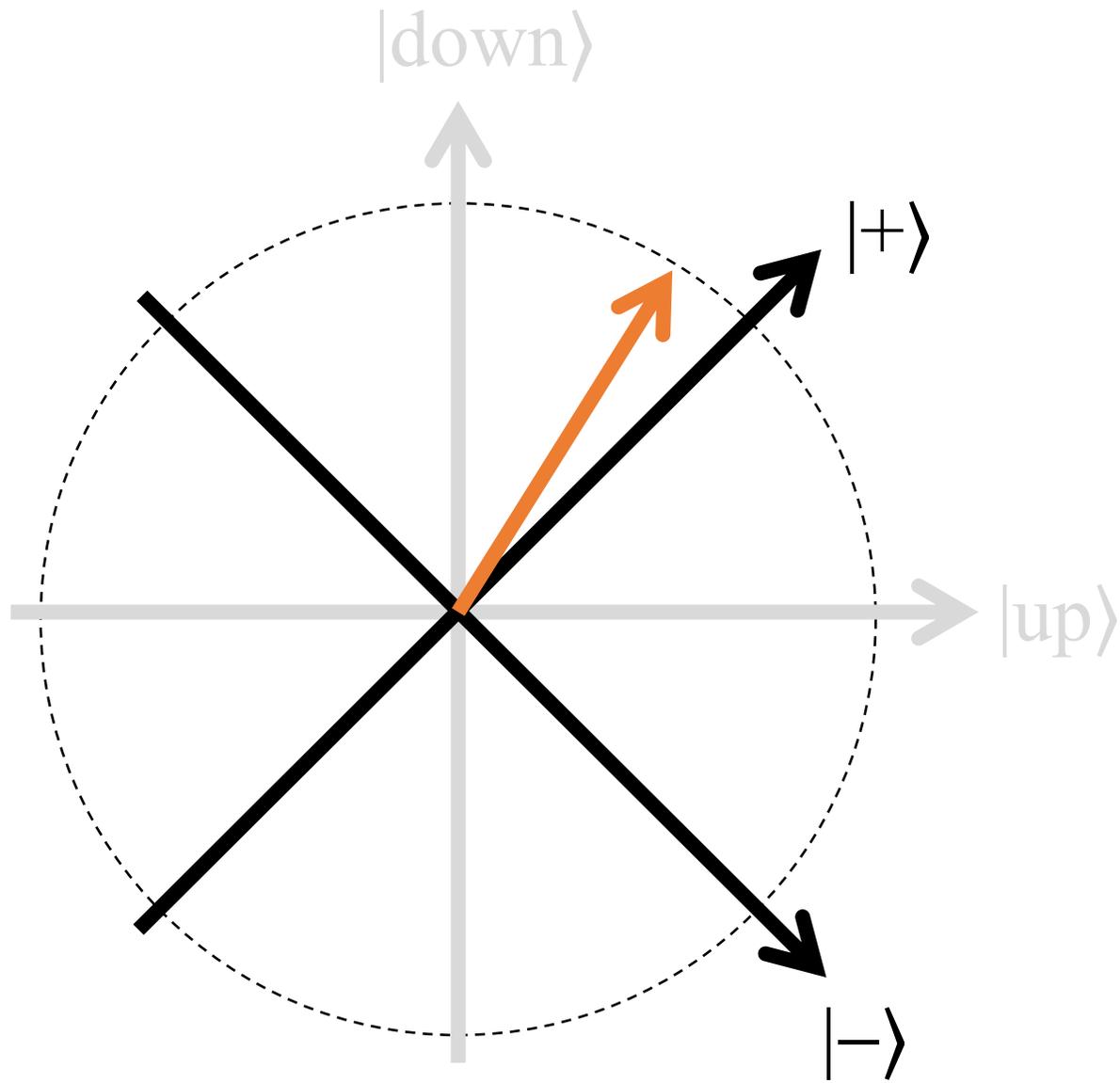


$|\text{down}\rangle$

The measurement problem

- The problem of **outcomes**:
How and why a single outcome is chosen?





The quantum state can be written in any basis in the Hilbert space

$$|+\rangle = |up\rangle + |down\rangle$$

$$|-\rangle = |up\rangle - |down\rangle$$

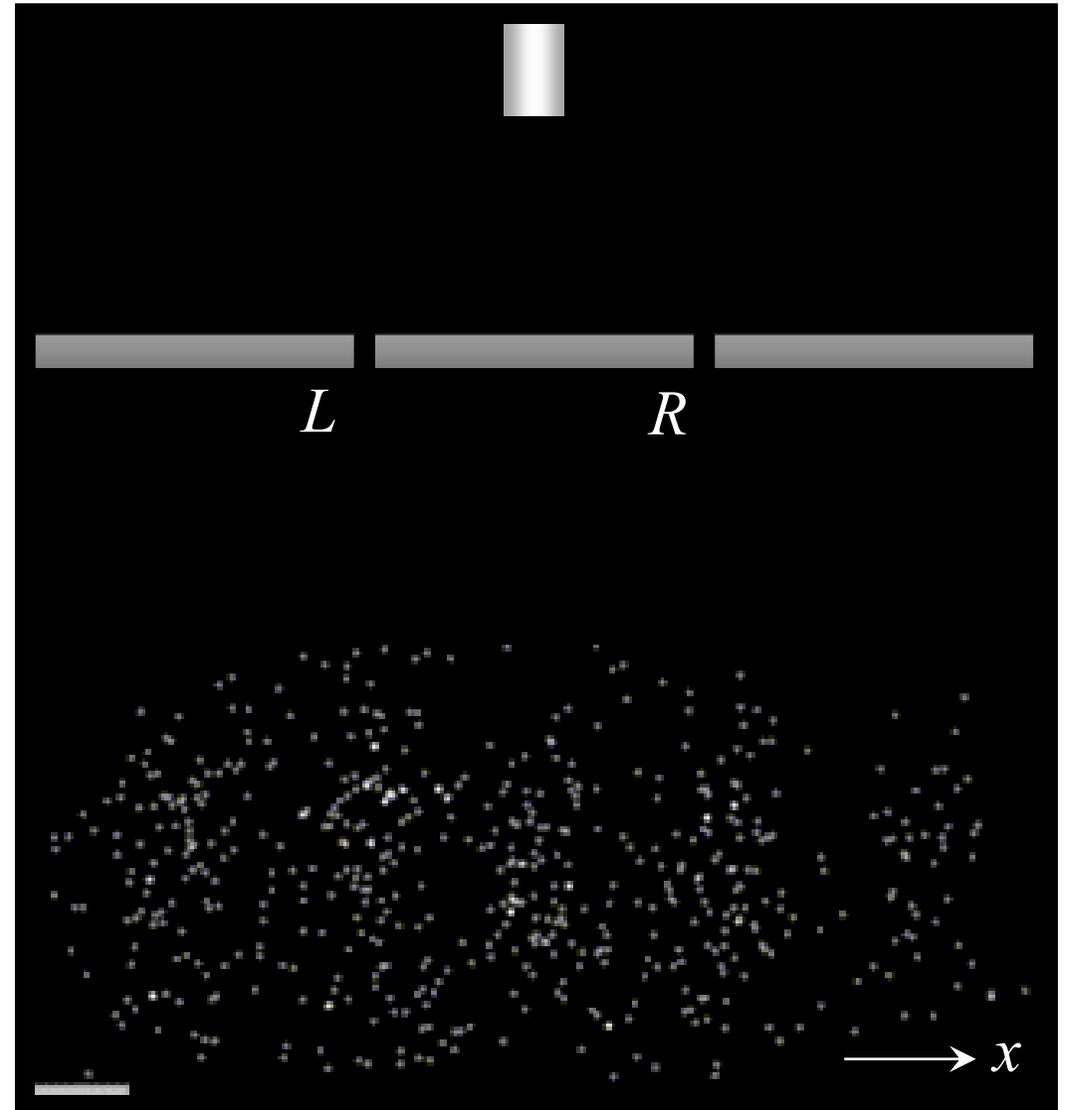
The measurement problem

- The problem of **outcomes**:
How and why a single outcome is chosen?
- The problem of **preferred basis**:
What singles out preferred outcomes in nature?

Double-slit experiment

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\psi_L\rangle + |\psi_R\rangle)$$

$$\begin{aligned} P(x) &= |\langle x|\Psi\rangle|^2 \\ &= \frac{1}{2}|\psi_L(x) + \psi_R(x)|^2 \\ &= \frac{1}{2}|\psi_L(x)|^2 + \frac{1}{2}|\psi_R(x)|^2 \\ &\quad + \text{Re}[\psi_L(x)\psi_R^*(x)] \end{aligned}$$



The measurement problem

- The problem of **outcomes**:
How and why a single outcome is chosen?
- The problem of **preferred basis**:
What singles out preferred outcomes in nature?
- The problem of **nonobservability of interference**:
Why don't we see superposition effects everywhere?

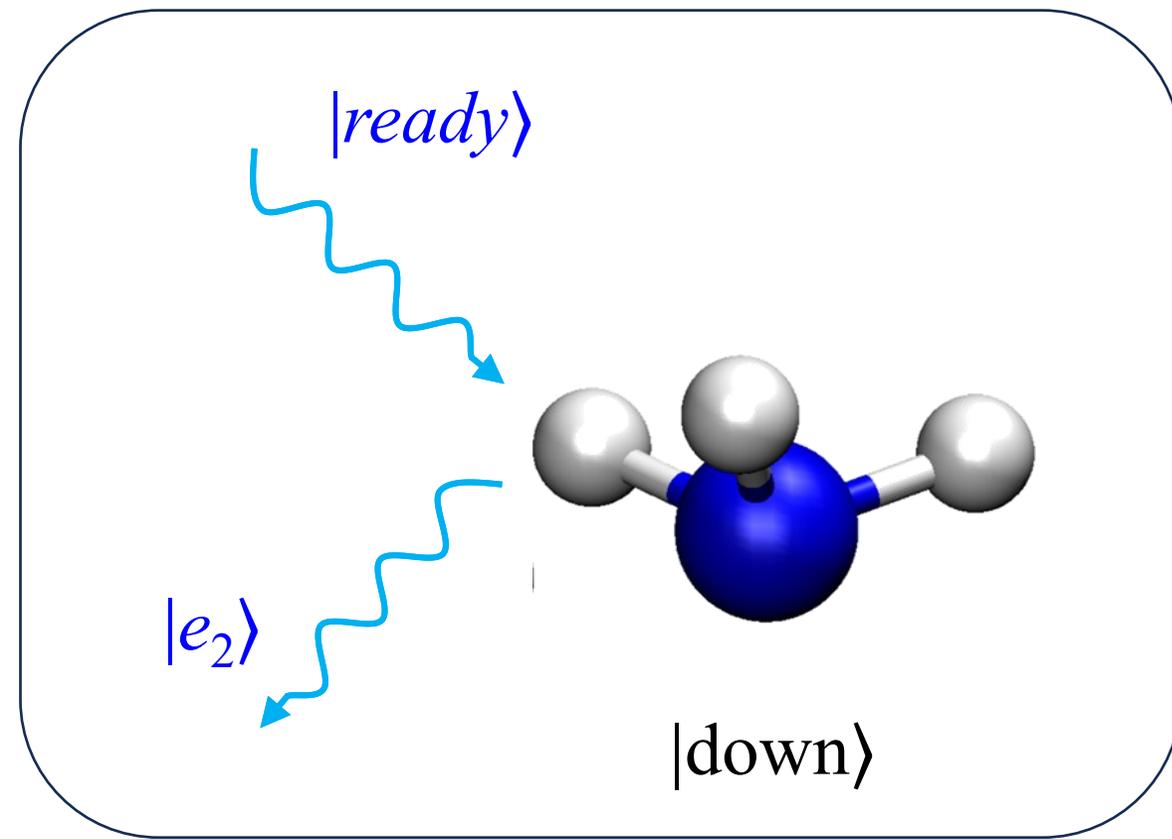
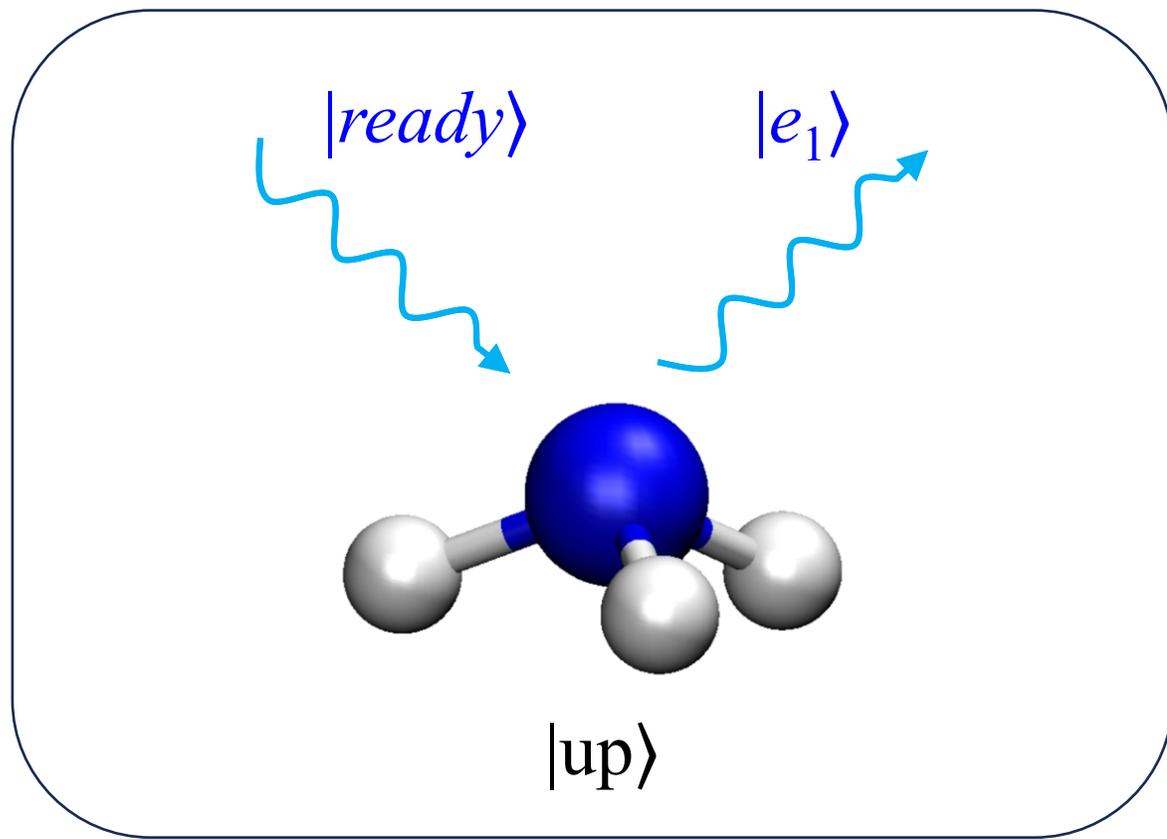
Decoherence: A quantum world in disguise

Decoherence transfers quantum information to the environment and suppresses the interference terms

$$\begin{aligned}
 & \begin{array}{ccc} & \text{System} & \text{Environment} \\ & \underbrace{\hspace{10em}} & \underbrace{\hspace{10em}} \end{array} \\
 |\Psi\rangle &= (a|up\rangle + b|down\rangle) \otimes |ready\rangle \\
 &= a|up\rangle|e_1\rangle + b|down\rangle|e_2\rangle
 \end{aligned}$$

System's reduced density matrix

$$\begin{aligned}
 \rho_S &= \text{Tr}_E [|\Psi\rangle\langle\Psi|] \\
 &= \begin{bmatrix} |a|^2 & ab^* \langle e_2|e_1\rangle \\ ba^* \langle e_1|e_2\rangle & |b|^2 \end{bmatrix}
 \end{aligned}$$



$$\langle e_1 | e_2 \rangle \rightarrow 0$$

$$\begin{aligned}
 & \begin{array}{ccc} & \text{System} & \text{Environment} \\ & \underbrace{\hspace{10em}} & \underbrace{\hspace{10em}} \end{array} \\
 |\Psi\rangle &= (a|up\rangle + b|down\rangle) \otimes |ready\rangle \\
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 &= \begin{bmatrix} |a|^2 & ab^* \langle e_2|e_1\rangle \\ ba^* \langle e_1|e_2\rangle & |b|^2 \end{bmatrix} \xrightarrow{\text{Decoherence}} \begin{bmatrix} |a|^2 & 0 \\ 0 & |b|^2 \end{bmatrix}
 \end{aligned}$$

Lindblad evolution

$$\rho_S = \text{Tr}_E[\rho]$$

$$\frac{d\rho_S}{dt} = -\frac{i}{\hbar}[\hat{H}, \rho_S] + \sum_k \Gamma_k \left(\hat{L}_k \rho_S \hat{L}_k^\dagger - \frac{1}{2} \{ \hat{L}_k^\dagger \hat{L}_k, \rho_S \} \right)$$

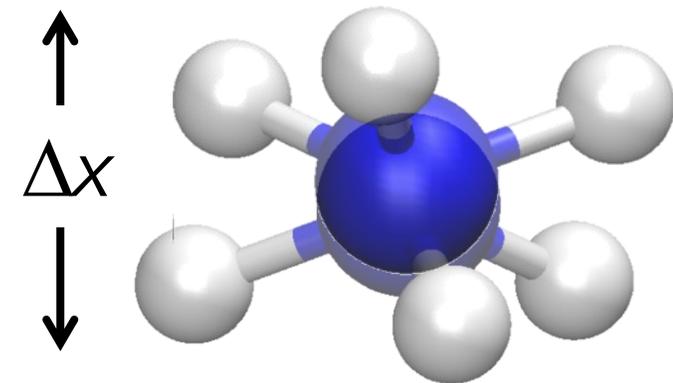
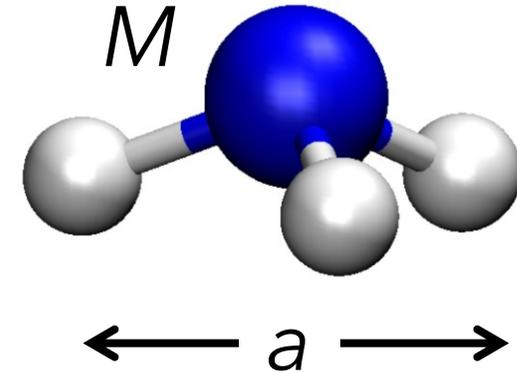
This term is responsible
for decoherence

Decoherence is fast!

Off-diagonal terms

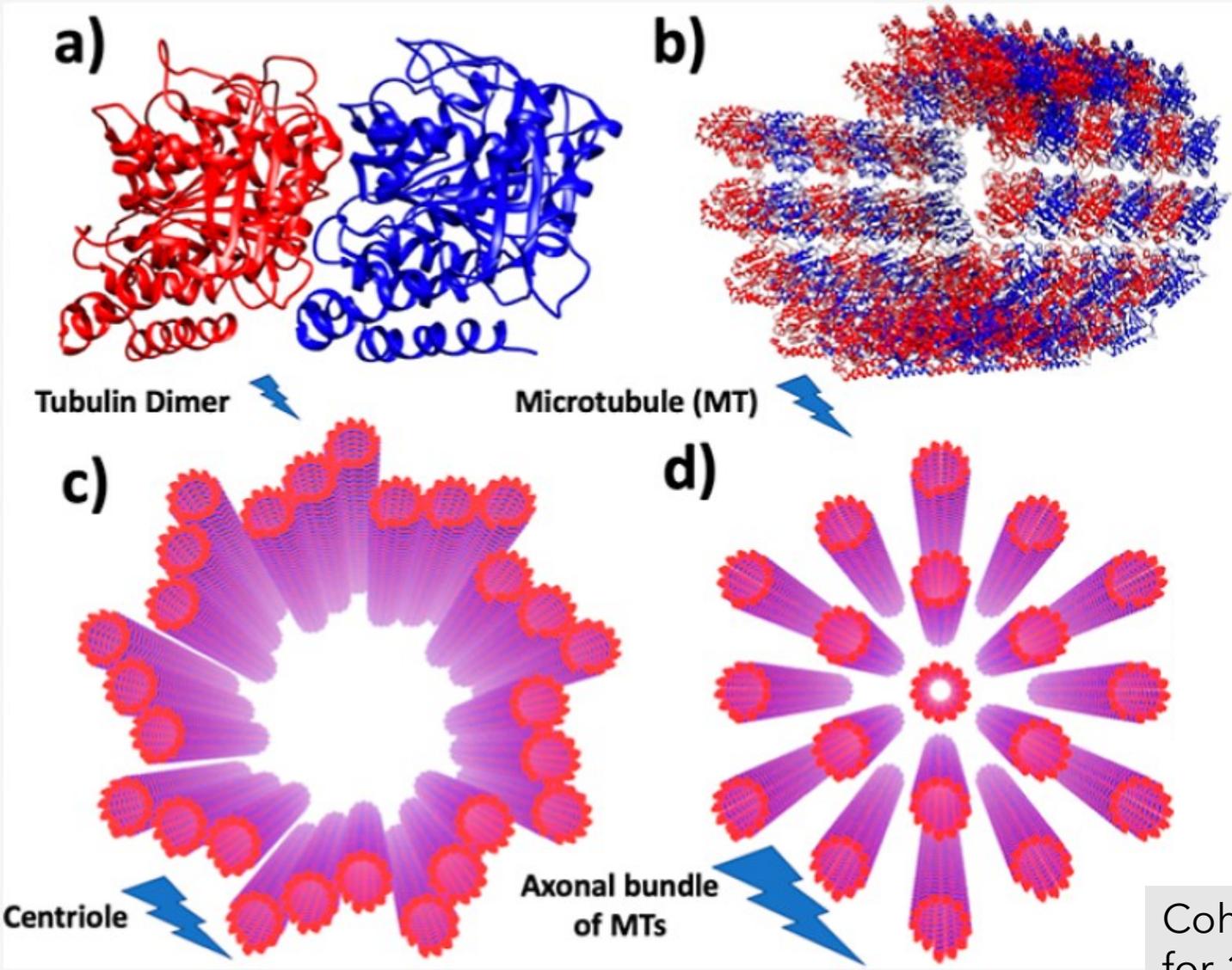
$$\rho_S(\text{up}, \text{down}, t) = \rho_S(\text{up}, \text{down}, 0) e^{-t/\tau_D}$$

$$\tau_D = \frac{1}{\frac{8}{3\hbar^2} \frac{N}{V} (2\pi M)^{1/2} a^2 \Delta x^2 (k_B T)^{3/2}}$$
$$= 10^{-19} \text{ s}$$



But sometimes,
decoherence may take longer

Superradiance in Trp protein architectures



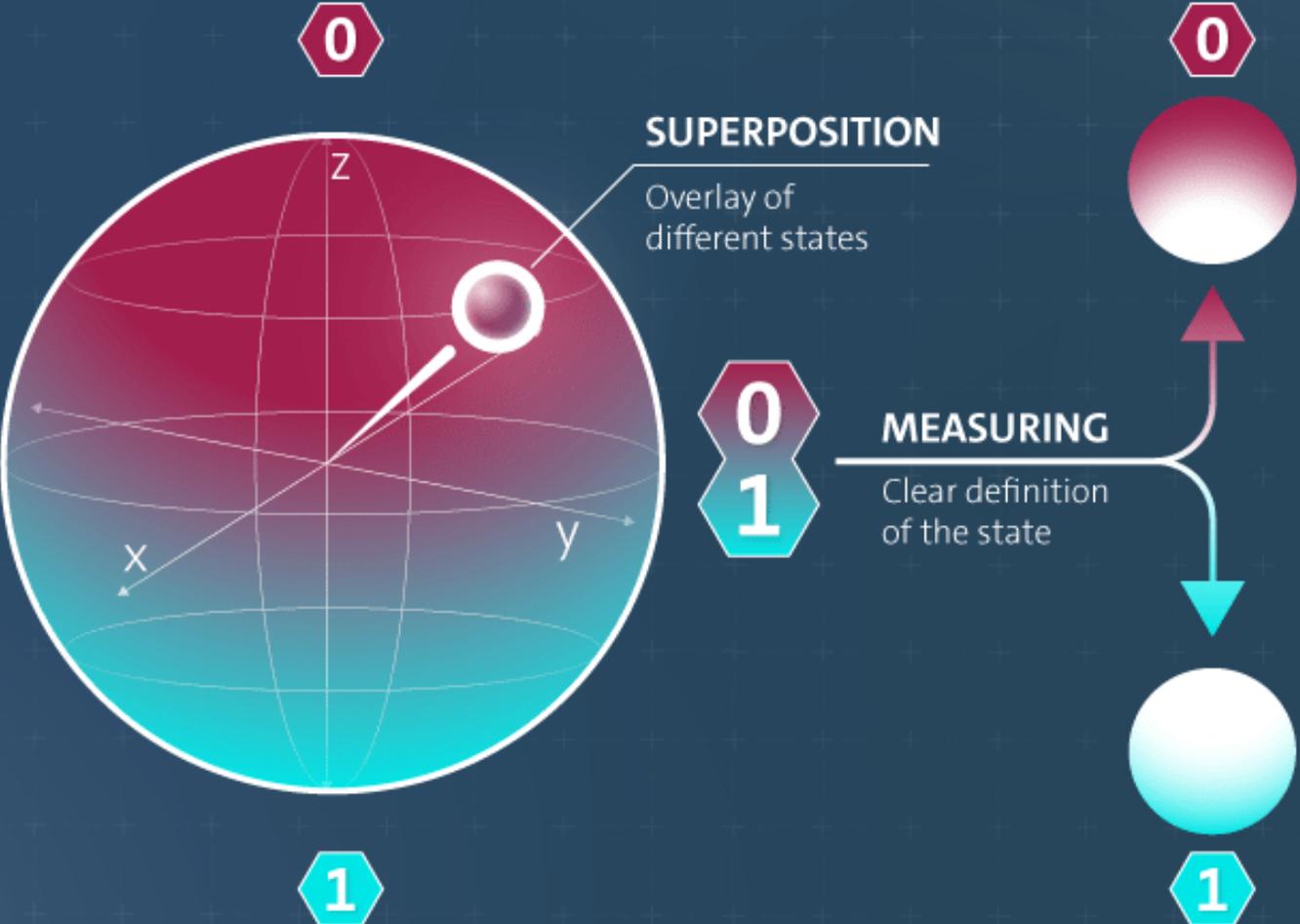
Coherences should live for 100 fs to explain the results

Long-lived coherences are the heart of quantum computing

Classical Bit
Binary system



quantum bit "qubit"
Arbitrarily manipulable two-state quantum system

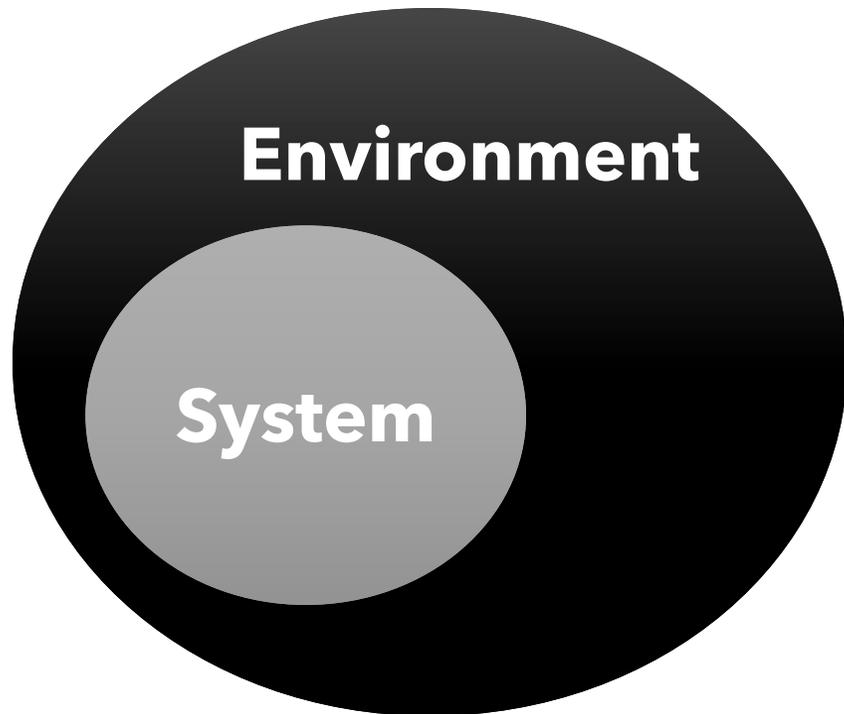


- Parallel arithmetic operations possible
- Exponential multiplication per qubit
- Massive amounts of data can be handled in plausible time

On which basis does decoherence occur?

Einselection:

Decoherence drives the quantum state to the basis **least entangled** with the environment



1. $\hat{H}_{system} \gg \hat{H}_{int}$

**Basis on which
decoherence occurs**

\hat{H}_{system} (energy) eigenstates

2. $\hat{H}_{system} \ll \hat{H}_{int}$

\hat{H}_{int} eigenstates (position)

3. $\hat{H}_{system} \approx \hat{H}_{int}$

Coherent states

Coherent states are quantum states of the harmonic oscillator that most closely resemble classical oscillations, thereby minimizing the uncertainty relation and achieving equal variances in position and momentum.

In a molecule, electrons lose coherence to nuclear vibrations because $\hat{H}_{elec} \gg \hat{H}_{vib}$

The *least entangled basis* is \hat{H}_{elec} eigenstates (**electronic energy**)

In a gas, a molecule loses coherence to the environment because $\hat{H}_{int} \gg \hat{H}_{mol}$

The *least entangled basis* is \hat{H}_{int} eigenstates (**molecular position**)

Decoherence solves:

- The problem of **preferred basis**:
What singles out preferred outcomes in nature?
- The problem of **nonobservability of interference**:
Why don't we see superposition effects everywhere?

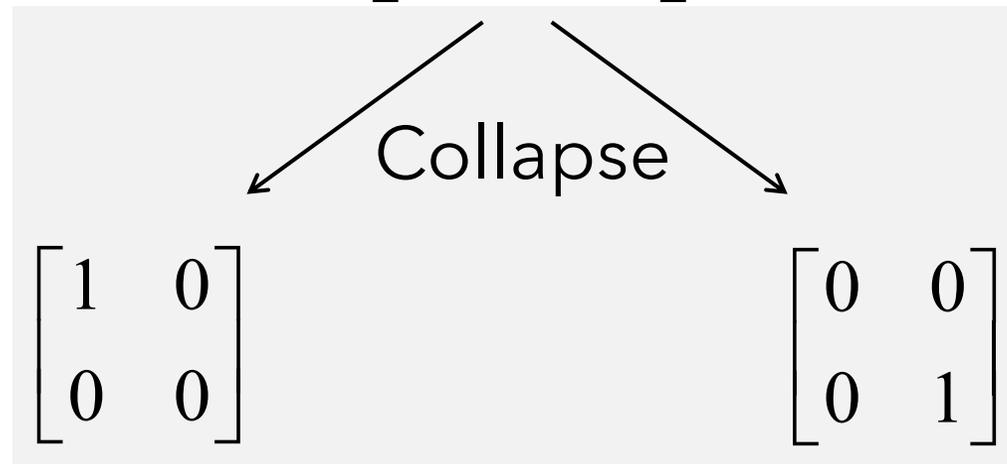
Decoherence does NOT solve:

- The problem of **outcomes**:
How and why a single outcome is chosen?

The problem of outcomes

$$\rho_S = \text{Tr}_E [\hat{\rho}]$$

$$= \begin{bmatrix} |a|^2 & ab^* \langle e_2 | e_1 \rangle \\ ba^* \langle e_1 | e_2 \rangle & |b|^2 \end{bmatrix} \xrightarrow{\text{Decoherence}} \begin{bmatrix} |a|^2 & 0 \\ 0 & |b|^2 \end{bmatrix}$$

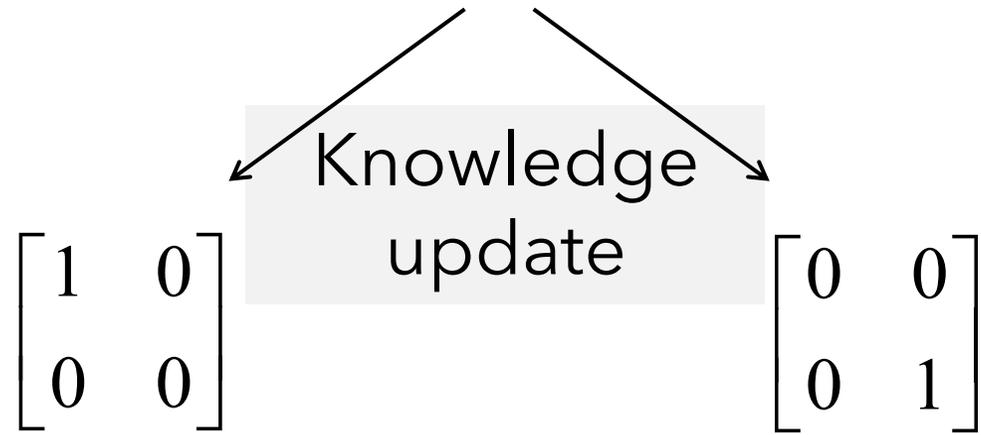


Collapse is not a consensus

QBism

$$\rho_S = \text{Tr}_S [\hat{\rho}]$$

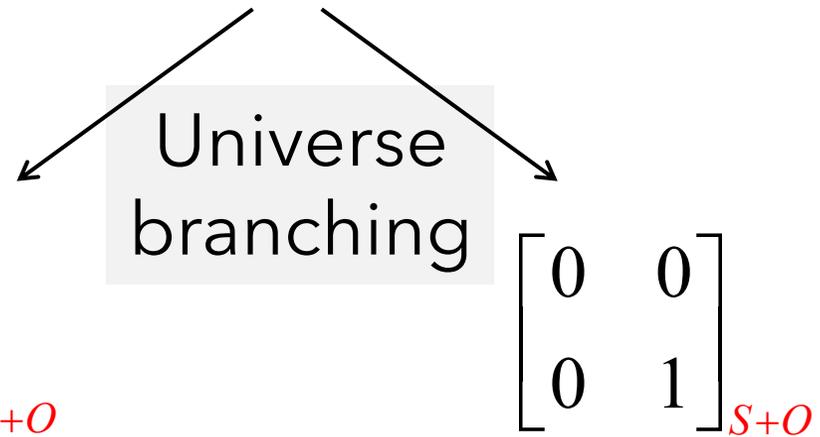
$$= \begin{bmatrix} |a|^2 & ab^* \langle e_2 | e_1 \rangle \\ ba^* \langle e_1 | e_2 \rangle & |b|^2 \end{bmatrix} \xrightarrow{\text{Decoherence}} \begin{bmatrix} |a|^2 & 0 \\ 0 & |b|^2 \end{bmatrix}_S$$



Many-worlds interpretation

$$\rho_S = \text{Tr}_S [\hat{\rho}]$$

$$= \begin{bmatrix} |a|^2 & ab^* \langle e_2 | e_1 \rangle \\ ba^* \langle e_1 | e_2 \rangle & |b|^2 \end{bmatrix} \xrightarrow{\text{Decoherence}} \begin{bmatrix} |a|^2 & 0 \\ 0 & |b|^2 \end{bmatrix}_S$$

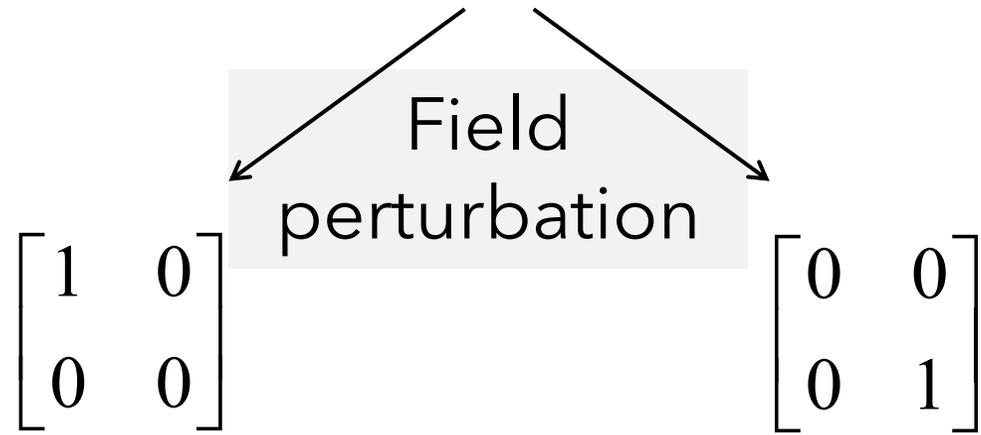




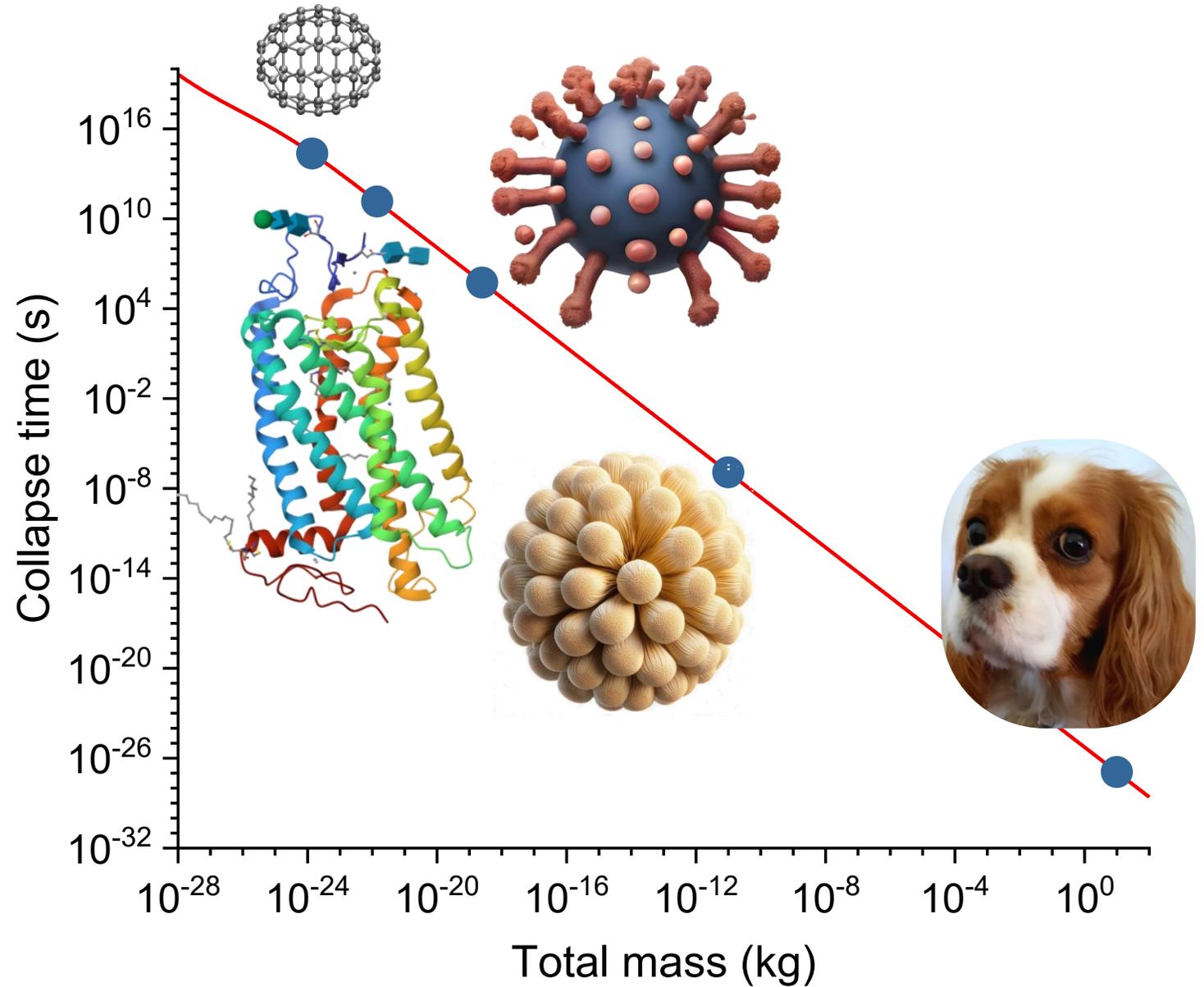
Objective collapse

$$\rho_S = \text{Tr}_S [\hat{\rho}]$$

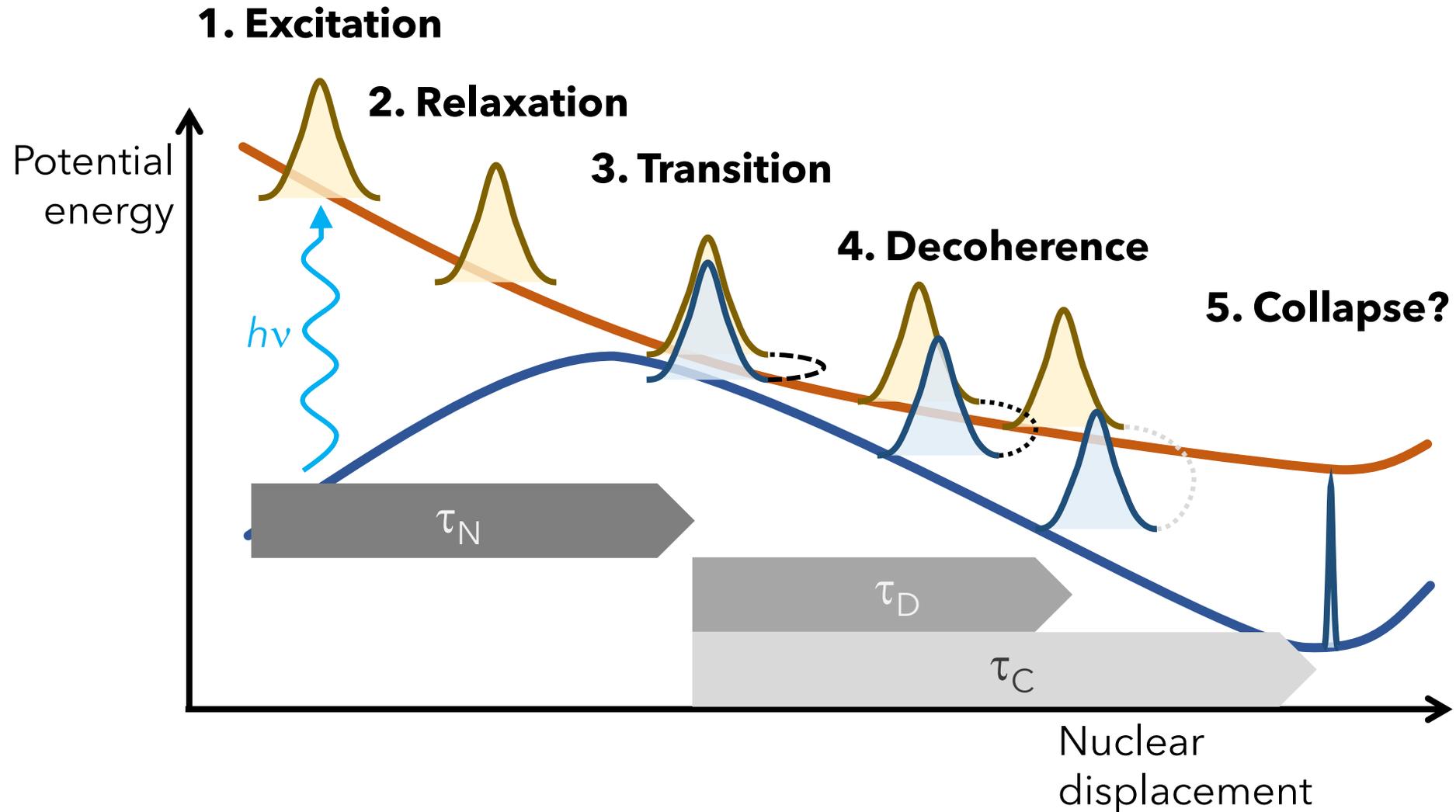
$$= \begin{bmatrix} |a|^2 & ab^* \langle e_2 | e_1 \rangle \\ ba^* \langle e_1 | e_2 \rangle & |b|^2 \end{bmatrix} \xrightarrow{\text{Decoherence}} \begin{bmatrix} |a|^2 & 0 \\ 0 & |b|^2 \end{bmatrix}_S$$



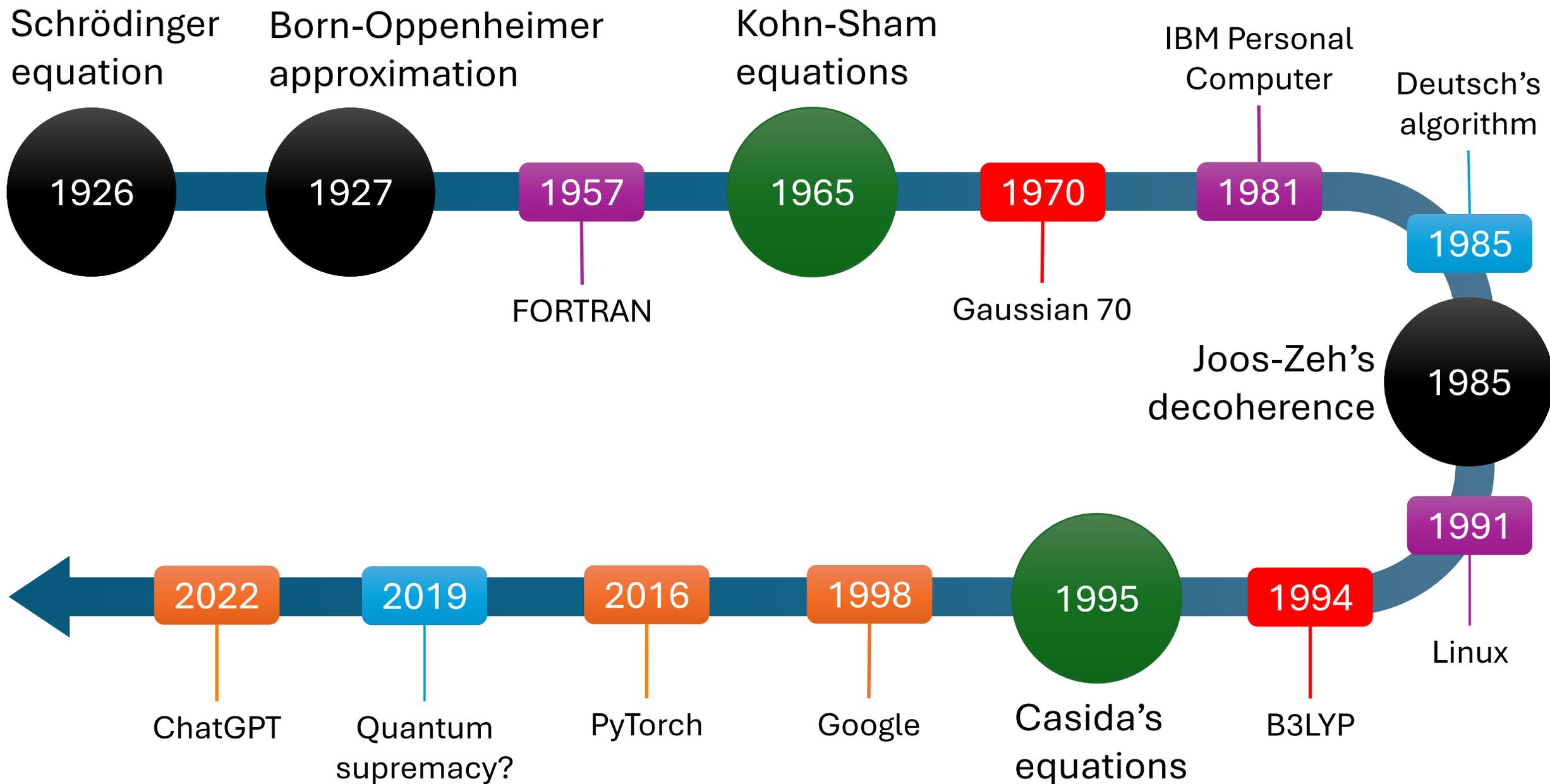
Diósi-Penrose gravity-induced collapse



A quasi-classical world



Quantum chemistry in context



To know more:

The measurement problem

- Tomaz; Mattos; Barbatti. *arXiv [quant-ph]* **2025**

Decoherence

- Hossenfelder, youtu.be/igsulul_HAQ
- Ball, Aeon Mag. **2017**, tinyurl.com/balldecoh
- Shu; Truhlar. *JCTC* **2023**, 19, 380

Insights into quantum chemistry

- Barbatti. *Pure Appl Chem* **2025**